

CS 61C
Fall 2018

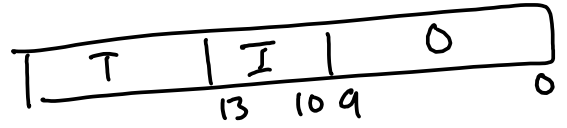
Caches II, Floating Point
Discussion 6: October 1, 2018

1 Code Analysis

Given the follow chunk of code, analyze the hit rate given that we have a byte-addressed computer with a total memory of 1 MiB. It also features a 16 KiB Direct-Mapped cache with 1 KiB blocks.

```
#define NUM_INTS 8192 // 2^13
int A[NUM_INTS]; // A lives at 0x10000
int i, total = 0;
for (i = 0; i < NUM_INTS; i += 128) {
    A[i] = i; // Line 1
}
for (i = 0; i < NUM_INTS; i += 128) {
    total += A[i]; // Line 2
}
```

$M: B = 2^{20} B$



1.1 How many bits make up a memory address on this computer?

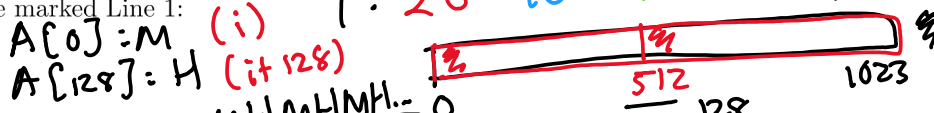
$\log_2 2^{20} = 20 \text{ bit address}$

1.2 What is the T:I:O breakdown?

O: $\log_2 2^{10} = 10 \text{ bits}$ I: $\log_2 \frac{16 \text{ KiB}}{1 \text{ KiB}} = \log_2 16 = 4 \text{ bits}$

1.3 Calculate the cache hit rate for the line marked Line 1:

$128 \cdot 4 \text{ B} = 512 \text{ B}$

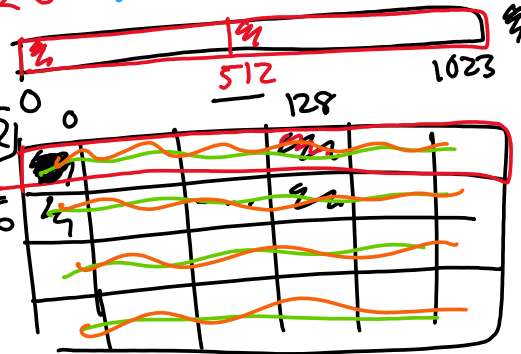


1.4 Calculate the cache hit rate for the line marked Line 2:

$2^{13} \text{ ints} \cdot \frac{4 \text{ B}}{\text{int}} = 2^{15}$

Cache: 2^{14}
2 AMAT

MHMH (same as above) $\Rightarrow 50\% \text{ HR}$



Recall that AMAT stands for Average Memory Access Time. The main formula for it is:

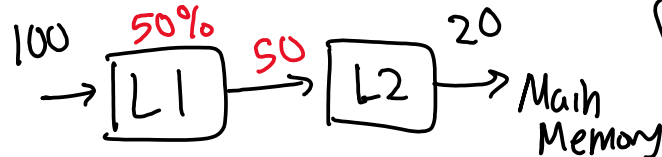
$AMAT = \text{Hit Time} + \text{Miss Rate} \cdot \text{Miss Penalty}$

We also have two types of miss rates, global and local. Global is calculated as: Fraction of ALL accesses that missed at that level over all accesses total. Whereas local is calculated: Fraction of ALL access that missed at that level over all access to that level total.

Second loop reverse
HH ... HH | MH ... MH
 $\frac{1}{2} \cdot 100\% + \frac{1}{2} \cdot 50\%$
75% HR

2.1 An L2\$, out of 100 total accesses to the cache system, missed 20 times. What is the global miss rate of L2\$?

$GMR = \frac{\text{misses here}}{\text{all mem accesses}}$
 $LMR = \frac{\text{misses here}}{\text{accesses here}}$



75% HR

$$GMR L2 = \frac{20}{100} = 20\%$$

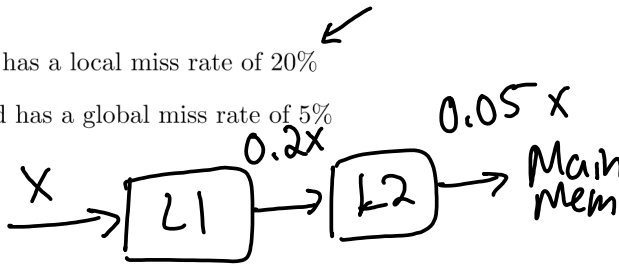
2.2 If L1\$ had a miss rate of 50%, what is the local miss rate of L2\$?

$$LMR L2 = \frac{20}{50} = 40\%$$

Suppose your system consists of:

1. An L1\$ that hits in 2 cycles and has a local miss rate of 20%
2. An L2\$ that hits in 15 cycles and has a global miss rate of 5%
3. Main memory hits in 100 cycles

2.3 What is the local miss rate of L2\$?



$$\frac{.05x}{.2x} = 25\%$$

$$LMR = \frac{GMR}{L1MR}$$

2.4 What is the AMAT of the system?

$$1.2 + 0.2(15 + 0.25(100)) = 2 + \frac{1}{5}(40) = 10$$

Handwritten annotations: L1HT, L1MR, L2HT, L2LMR, Main Mem

2.5 Suppose we want to reduce the AMAT of the system to 8 cycles or lower by adding in a L3\$. If the L3\$ has a local miss rate of 30%, what is the largest hit time that the L3\$ can have?

$$1.2 + \frac{1}{5} \cdot 15 + \frac{1}{20} \cdot 100 = 2 + 3 + 5 = 10$$

3 Floating Point

The IEEE 754 standard defines a binary representation for floating point values using three fields:

- The *sign* determines the sign of the number (0 for positive, 1 for negative)
- The *exponent* is in **biased notation** with a bias of 127
- The *significand or mantissa* is akin to unsigned, but used to store a fraction instead of an integer

The below table shows the bit breakdown for the single precision (32-bit) representation.

32

1	8	23
Sign	Exponent	Mantissa/Significand/Fraction

For normalized floats:

$$\text{Value} = (-1)^{\text{Sign}} * 2^{\text{Exp}-\text{Bias}} * 1.\text{significand}_2$$

For denormalized floats:

$$\text{Value} = (-1)^{\text{Sign}} * 2^{\text{Exp}-\text{Bias}+1} * 0.\text{significand}_2$$

Cheat Sheet

Exponent	Significand	Meaning
0	Anything	Denorm
1-254	Anything	Normal
255	0	Infinity
255	Nonzero	NaN

Handwritten notes and diagrams for IEEE 754 float representation:

- Diagram of a float: $\text{sign} \cdot 1.\text{mantissa} \times 2^{\text{exp}}$
- Diagram of a denormalized float: $\text{sign} \cdot 0.\text{mantissa} \times 2^{\text{exp}-\text{bias}+1}$
- Bit patterns: $0 \text{ ||| ||| ||| } 00 \dots 00$ and $0 \text{ ||| ||| ||| } 0 \dots 1.00$
- Label "implied" pointing to the leading 1 in the mantissa.

3.1 How many zeroes can be represented using a float?

$$00 \dots 00 \leftarrow 0's$$

$$10 \dots 00 \leftarrow 0's$$

0x7F7FFFFF

3.2 What is the largest finite positive value that can be stored using a single precision float?

0 111...111 → NaN

$1,111...11 \times 2^{254-127} = 1,111...11 \times 2^{127}$
 $= \frac{1,111...11}{2} \times 2^{127}$
 $(2 - 2^{-23}) \times 2^{127}$
 10.000...

3.3 What is the smallest positive value that can be stored using a single precision float?

3.4 What is the smallest positive normalized value that can be stored using a single precision float?

$0.5 \quad 0.25$
 $2^{-1} \quad 2^{-2}$
 $\downarrow \quad \downarrow$
 $2^{-3} \quad 2^{-4} \dots$

3.5 Cover the following numbers from binary to decimal or from decimal to binary:

- 0x00000000
- 39.5625
- 8.25 (1)
- 0xFF94BEEF
- 0x00000F00
- $-\infty$

(1) 1000.0100
 1.00001×2^3
 0 10000010 000010...0

$3 + 127 = 130$
 0x41040000

4 Extra Stuff on Caches!

4.1 Heres some practice involving a 2-way set associative cache. This time we have an 8-bit address space, 8 B blocks, and a cache size of 32 B. Classify each of the following accesses as a cache hit (H), cache miss (M) or cache miss with replacement (R). For any misses, list out which type of miss it is.

Address	T/I/O	Hit, Miss, Replace
0b0000 0100		
0b0000 0101		
0b0110 1000		
0b1100 1000		
0b0110 1000		
0b1101 1101		
0b0100 0101		
0b0000 0100		
0b1100 1000		

4.2 What is the hit rate of our above accesses?