Lab 07 WS

Friday, July 5, 2019 12:25 PM

CS 61BL Summer 2019 — Lab 7 — July 5, 2019

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Please complete this worksheet during your lab, and turn it in to your TA by the end of your section. You are encouraged to work with your neighbors collaboratively.

Section Number: (01) (02) (03) (04) (05) (06) (07) (08) (09) (10) (11) (12)

1 Timing

1.1 Regarding the results of `Timer.java`, why are there differences between some students numbers and other students numbers? Why is it that the amount of time it takes to sort the same number of elements isnt always the same? What might contribute to these differences?

processor speed, other processes

2 Counting

2.1 Consider an `if ... else` of the form,

```
1 if (A) {
2     B;
3 } else {
4     C;
5 }
```

a+b

if (A) { } a+c

else { C }

where `A`, `B`, and `C` are program segments. (`A` might be a method call, for instance.)

Let a be defined as the number of steps it takes to evaluate `A`, b be that for `B`, and c be that for `C`. How many steps does it take to evaluate the entire `if ... else` block in terms of a, b, c? *Assume that* $a + b < c$.

Best case: a+b

Worst case: a+c

3 Using the Right Bounds

3.1 Copy the statement below, replacing "(your name here)" with your name, and add your signature acknowledging your understanding of the "Best Case and Worst Case" section of the spec.

I, (your name here), agree that Big Omega is not the same as Best Case and Big O is not the same as Worst Case.

No

Your Signature:

4 Analyzing Functions

Provide the tightest bound possible for each of these functions, in terms of the function parameter `n` or in terms of the length of the `array` (which you may also call `n`).

4.1 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1 public void f1(int[] array) {
2     for (int i = 0; i < array.length; i++) {
3         for (int j = 0; j < Integer.MAX_VALUE; j++) {
4             System.out.println("Hi!");
5         }
6     }
7 }
```

n

constant

No constant

Answer: $\Theta(n)$

$\Omega(n)$ $O(n)$

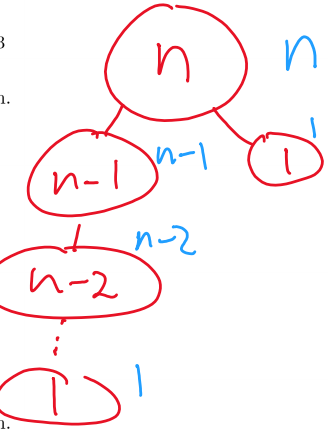
size
work

4.2 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1  public int f2(int n) {
2      if (n <= 1) return n;
3      f1(new int[n]);
4      return n + n * f2(n - 1) + n * n * f2(1);
5  }
```

Answer: $\Theta(n^2)$

$n + (n-1) + (n-2) + \ldots + 1 = \frac{n(n-1)}{2}$

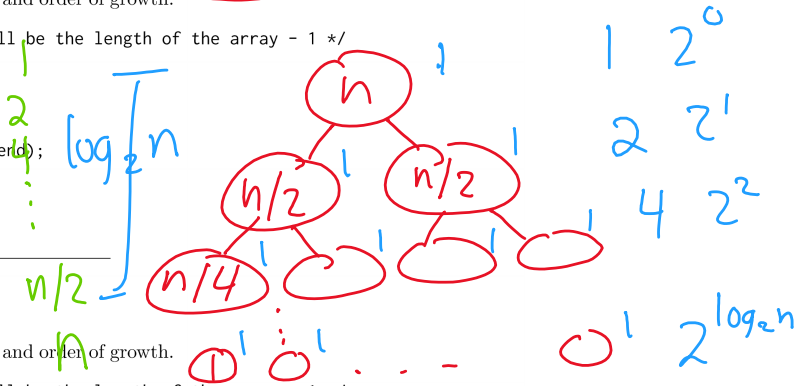


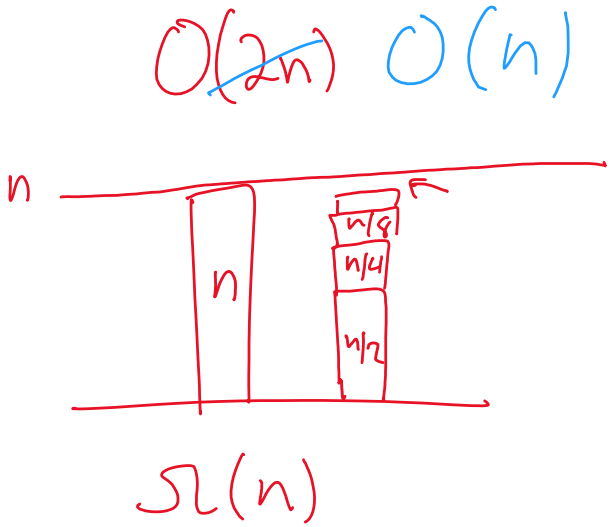
4.3 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1  /* When f3 is first called, start will be 0 and end will be the length of the array - 1 */
2  public int f3(char[] array, int start, int end) {
3      if (array.length <= 1 || end <= start) return 1;
4      int mid = start + ((end - start) / 2);
5      return f3(array, start, mid) + f3(array, mid + 1, end);
6  }
```

$n + \frac{n}{2} + \frac{n}{4} + \ldots + 4 + 2 + 1$

Answer: $\Theta(n)$





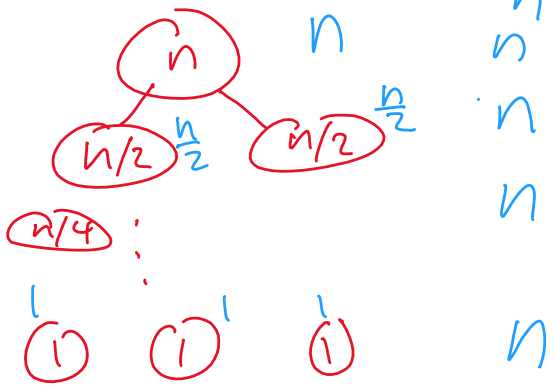
4.4 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1  /* When f4 is first called, start will be 0 and end will be the length of the array - 1 */
2  public int f4(char[] array, int start, int end) {
3      if (array.length <= 1 || end <= start) return 1;
4      int counter = 0;
5      for (int i = start; i < end; i++) {
6          if (array[i] == 'a') counter++;
7      }
8      int mid = start + ((end - start) / 2);
9      return counter + f4(array, start, mid) + f4(array, mid + 1, end);
10 }
```

Answer: $\Theta(n \cdot \log n)$

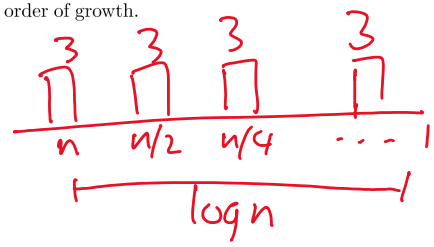
n work/layer

log n layers



4.5 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1 public void f5(int n) {
2     int[] array = {1, 2, 3};
3     while (n > 0) {
4         f1(array);
5         n = n / 2;
6     }
7 }
```

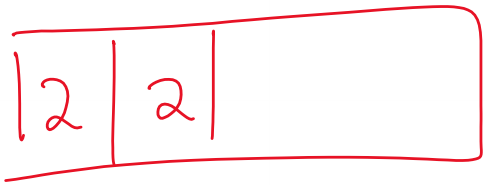


Answer: $O(\log n)$ $O(3 \log n)$

4.6 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1 public void f6(int[] array) {
2     for (int i = 1; i < array.length; i++) {
3         if (array[i] == array[i-1]) {
4             System.out.println("Sarah is the potatoest");
5             return;
6         }
7     }
8 }
```

Answer: $\Omega(1), O(N)$



best $O(1)$

worst: $O(N)$

4.7 What is the runtime? Provide the appropriate bound notation and order of growth.

```
1  /* When f7 is first called, start will be 0 and end will be the length of the array - 1 */
2  public int f7(int[] array, int start, int end) {
3      if (array.length <= 1 || end <= start) {
4          return 1;
5      } else if (array[start] <= array[end]) {
6          return f7(array, start + 1, end - 1);
7      } else {
8          int tmp = array[start];
9          array[start] = array[end];
10         array[end] = tmp;
11         return f7(array, start + 1, end) + f7(array, start, end - 1)
12                 + f7(array, start + 1, end - 1);
13     }
14 }
```

Answer(s): $\Omega(n), O(3^n)$

4.8 For `f7` in the previous subquestion, instead of asking "What is the runtime", what if we asked you "What is the best and worst case runtime?" instead?

Best Case: $O(n)$ Worst Case: $O(3^n)$

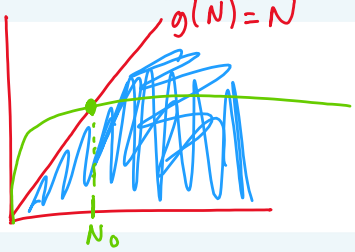
Big-O Definition 1

We say that

$\log n \in O(N)$

$$f(N) \in O(g(N))$$

iff $\exists\, k_2 > 0$ s.t. $f(N) \leq k_2 g(N) \quad \forall\, N > N_0.$

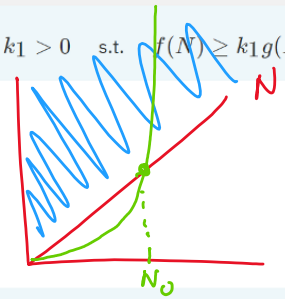


Big-Omega Definition

We say that

$$f(N) \in \Omega(g(N))$$

iff $\exists\, k_1 > 0$ s.t. $f(N) \geq k_1 g(N) \quad \forall\, N > N_0.$

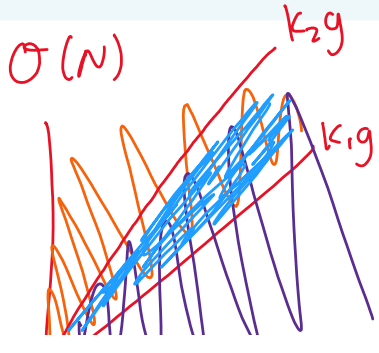


Big-Theta Definition 1

We say that

$$f(N) \in \Theta(g(N))$$

iff $\exists\, k_1, k_2 > 0$ s.t. $k_1 g(N) \leq f(N) \leq k_2 g(N) \quad \forall\, N > N_0.$



$$\Theta(50N) \rightarrow \Theta(N)$$

$$\Omega(N) + O(N) \Rightarrow \Theta(N)$$